These values in (1) give us

$$c\log\left(\frac{n\pm\sqrt{(n^2-4c^2)}}{2c}\right)-m\mp\frac{n}{4c}\sqrt{(n^2-4c^2)}=0,$$

for the equation to the evolute.

136. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa.

Evaluate the definite integral

$$\int_0^1\int_0^1\frac{v^{l-1}u^{m-1}(1-v^n)^{p-1}(1-u^s)^{r-1}dvdu}{[bv^n+c(1-v^n)]^{p+l/n}(u^s+a)^{r+m/s}}.$$

Solution by the PROPOSER.

Let $u^n=z$, $u^s=w$, A=value of integral.

$$\therefore A=\int_0^1\int_0^1\frac{z^{l/n-1}w^{m/s-1}(1-z)^{p-1}(1-w)^{r-1}dzdw}{[bz+c(1-z)]^{p+l/n}(w+a)^{r+m/s}}.$$

$$\text{Let } z=\frac{cy}{b(1-y)+cy},\quad \frac{w}{w+a}=\frac{x}{1+a}.$$

$$\therefore A=\frac{1}{a^r(1+a)^{m/s}b^{l/n}c^p}\int_0^1\int_0^1 y^{l/n-1}x^{m/s-1}(1-y)^{p-1}(1-x)^{r-1}dydx$$

$$=\frac{1}{a^r(1+a)^{m/s}b^{l/n}c^p}\cdot\frac{\Gamma(l/n)\Gamma(p)\Gamma(m/s)\Gamma(r)}{\Gamma(l/n+p)\Gamma(m/s+r)}.$$

MECHANICS.

133. Proposed by J. C. CORBIN, Superintendent of Schools, Pine Bluff, Ark.

A stick of square-edged timber is 20 feet long, 10 inches square at large end, and 6 inches square at small end. How far from either end must a hand spike be placed so that two men with the hand spike and one man at the end shall each have an equal weight to carry?

Solution by G. B. M. ZERR, A. M., Ph. D., The Temple College, Philadelphia, Pa., and P. H. PHILBRICK, C. E., Lake Charles, La.

Let H=height of pyramid, base 10 inches square; h=height of pyramid, base 6 inches square; z, z_1, z_2 the distances of the centers of mass of the frustum, large pyramid and small pyramid, respectively, from the larger base; m, n the masses of the two pyramids.

$\therefore mz_1=nz_2+(m-n)z$, $z_1=\frac{1}{4}H$, $z_2=\frac{1}{4}h+H-h=H-\frac{3}{4}h$.

Also the masses are to each other as the cubes of the heights.

$\therefore \frac{1}{4}H^4=h^3(H-\frac{3}{4}h)+z(H^3-h^3)$.

$$\therefore z=\frac{H-h}{4}\left(\frac{H^2+2Hh+3h^2}{H^2+Hh+h^2}\right).$$

Let A, a be the lower and upper bases.

$$\therefore z=\tfrac{1}{4}(H-h)\left(\frac{A+2\sqrt{(Aa)}+3a}{A+\sqrt{(Aa)}+a}\right).$$

$H-h=20$ feet, $A=\frac{100}{144}=\frac{25}{36}$ square feet, $a=\frac{36}{144}=\frac{1}{4}$ square foot.

$\therefore z=8\frac{18}{49}$ feet. $20-8\frac{18}{49}=11\frac{31}{49}$.

Taking moments about the center of mass, $2x=11\frac{31}{49}$, $x=5\frac{40}{49}$, $8\frac{18}{49}-5\frac{40}{49}=2\frac{27}{49}$ feet from the larger end.

$2y=8\frac{18}{49}$, $y=4\frac{9}{49}$, $11\frac{31}{49}-4\frac{9}{49}=7\frac{22}{49}$ feet from the smaller end.

134. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

If $pv=Rt-b/tv$ be the equation for CO_2 gas, find the total, external and internal work done in compressing the gas from 102 to 136 atmospheres at a constant temperature 16°C, and constant volume, $R=63.23$, $b=481600$ for CO_2.

Solution by the PROPOSER.

136 atmospheres$=2000$ lbs.$=p_2$, 102 atmospheres$=1500$ lbs.$=p_1$.

$$pv=Rt-b/tv,\text{ or } v=\frac{Rt}{2p}+\frac{1}{2p}\sqrt{\frac{R^2t^3-4bp}{t}},\quad \frac{dv}{dt}=\frac{R}{2p}+\frac{R^2t^3+2bp}{2pt\sqrt{(R^2t^4-4bpt}}$$

$$=\frac{R}{2p}+\frac{b}{t\sqrt{(R^2t^4-4bpt)}}+\frac{R^2t^2}{2p\sqrt{(R^2t^4-4bpt)}}.$$

$$\text{External work}=t\int_{p_1}^{p_2}\left(\frac{dv}{dt}\right)dp$$

$$=t\int_{p_1}^{p_2}\left(\frac{R}{2p}+\frac{b}{t\sqrt{(R^2t^4-4bpt)}}+\frac{R^2t^2}{2p\sqrt{(R^2t^4-4bpt)}}\right)dp$$

$$=Rt\log_e\left(\frac{Rt^2-\sqrt{(R^2t^4-4bp_2t)}}{Rt^2-\sqrt{(R^2t^4-4bp_1t)}}\right)+\frac{1}{2t}[\sqrt{(R^2t^4-4bp_1t)}-\sqrt{(R^2t^4-4bp_2\mathrm{t})}].$$

Now $t=273+17=290$, $R=63.23$, $b=481600$.

$\therefore$ External work$=18336.7\log_e(1.3367)+46.087=5367.47$ ft. lbs.

$$\text{Total work}=\int_{p_1}^{p_2}vdp=\int_{p_1}^{p_2}\left(\frac{Rt}{2p}+\frac{1}{2p}\sqrt{\frac{R^2t^3-4bp}{t}}\right)dp$$

$$=Rt\log_e\left(\frac{Rt^2-\sqrt{(R^2t^4-4bp_2t)}}{Rt^2-\sqrt{(R^2t^4-4bp_1t)}}\right)+\frac{1}{t}[\sqrt{(R^2t^4-4bp_2t)}-\sqrt{(R^2t^4-4bp_1t)}].$$